INCLUSION OF INTERFACIAL SHEAR LAG EFFECT IN IMPEDANCE MODELS

INTRODUCTION

The piezo-impedance transducers are bonded to the surface of the host structures using an adhesive mix (such as epoxy), which forms a permanent finite thickness interfacial layer between the structure and the patch. In the analysis presented so far in this thesis, the effects of this layer were neglected. The force transmission from the PZT patch to the host structure was assumed to occur at the ends of the patch (1D model of Liang et al., 1994) or along the continuous boundary edges of the patch (2D effective impedance model, Chapter 5). In reality, the force transfer takes place through the interfacial bond layer via shear mechanism. This chapter reviews the mechanism of force transfer through the bond layer and presents a step-by-step derivation to integrate this mechanism into impedance formulations, both 1D and 2D. The influence of various parameters (associated with the bond layer) on the electro-mechanical admittance response are also investigated.

SHEAR LAG EFFECT

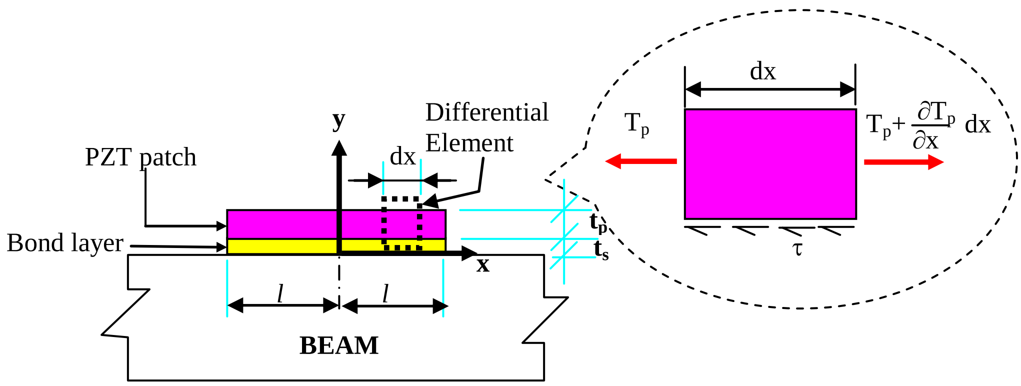


Fig. 1 A PZT patch bonded to a beam using adhesive bond layer.

Crawley and de Luis (1987) and Sirohi and Chopra (2000b) respectively modelled the actuation and sensing of a generic beam element using an adhesively bonded PZT patch. The typical configuration of the system is shown in Fig. 1. The patch has a length $2l$, width w_p and thickness t_p, while the bonding layer has a thickness equal to t_s. The adhesive layer thickness has been shown exaggerated to facilitate visualization. The beam has depth t_b and width w_b. Let T_p denote the axial stress in the PZT patch and τ the interfacial shear stress. Following assumptions were made by Crawley and de Luis (1987) and Sirohi and Chopra (2000b) in their analysis:

(i) The system is under quasi-static equilibrium.

(ii) The beam is actuated in pure bending mode and the bending strain is linearly distributed across any cross section.

(iii) The PZT patch is in a state of pure 1D axial strain.

(iv) The bonding layer is in a state of pure shear and the shear stress is independent of 'y'.

(v) The ends of the segmented PZT actuator/ sensor are stress free, implying a uniform strain distribution across the thickness of the patch.

A more detailed deformation profile is shown in Fig. 2, which shows the symmetrical right half of the system of Fig. 1. Let 'u_p' be the displacement at the interface between the PZT patch and the bonding layer and 'u' the corresponding displacement at the interface between the bonding layer and the beam.

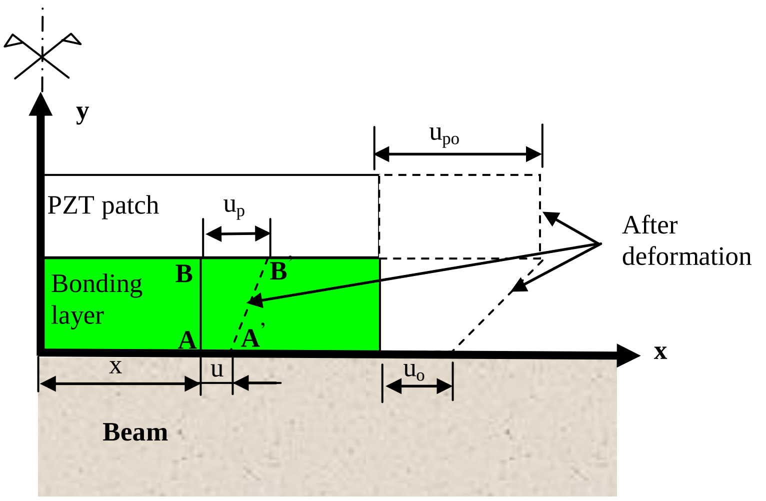


Fig. 2 Deformation in bonding layer and PZT patch.

PZT Patch as Sensor

Let the PZT patch be instrumented only to sense strain on the beam surface and hence no external electric field be applied across it. Considering the static equilibrium of the differential element of the PZT patch in the x-direction, as shown in Fig. 1, we can derive

$$\tau = \frac{\partial T_p}{\partial x} t_p \tag{1}$$

At any cross section of the beam, within the portion containing the PZT patch, the bending moment is given by

$$M = T_p w_p t_p (0.5t_b + t_s + 0.5t_p) \tag{2}$$

Also, from Euler-Bernoulli's beam theory,

$$M = -\sigma_b \left(\frac{I}{0.5t_b} \right) \tag{3}$$

where σ_b is the bending stress at the extreme fibre of the beam and 'I' the second moment of inertia of the beam cross-section. The negative sign signifies that sagging moment and tensile stresses are considered positive. Comparing Eqs. (2) and (3) and with $I = w_b t_b^3 / 12$, we get

$$\sigma_b + \left(\frac{3T_p w_p t_p}{w_b t_b^2} \right) (t_b + t_p + 2t_s) = 0 \tag{4}$$

Assuming $(t_p+2t_s) << t_b$,differentiating with respect to x, and substituting Eq. (1), we get

$$\frac{\partial \sigma_b}{\partial x} + \left(\frac{3w_p}{w_b t_b} \right) \tau = 0 \tag{5}$$

Further, from Hooke's law,

$$\sigma_b = Y_b S_b \tag{6}$$

$$T_p = Y^E S_p \tag{7}$$

$$\tau = G_s \gamma \tag{8}$$

where Y_b and Y^E respectively denote the Young's modulus of elasticity of the beam and the PZT patch (at zero electric field for the patch) respectively and S_b and S_p the corresponding strains. G_s denotes the shear modulus of elasticity of the bonding layer and γ the shear strain undergone by it. Substituting Eqs. (6) to (8) into Eqs. (1) and (5), we get Eqs. (9) and (10) respectively.

$$G_s\gamma = Y_p t_p \frac{\partial S_p}{\partial x} \tag{9}$$

$$Y_b \frac{\partial S_b}{\partial x} + \left(\frac{3w_p}{w_b t_b}\right) G_s \gamma = 0 \tag{10}$$

From Fig. 2, the shear strain in the bonding layer can be determined as

$$\gamma = \frac{u_p - u}{t_s} \tag{11}$$

Substituting Eq. (11) into Eqs. (9) and (10), differentiating with respect to x, and simplifying, we get Eqs. (12) and (13) respectively

$$\frac{\partial^2 S_p}{\partial x^2} = \left(\frac{G_s S_b}{Y_p t_s t_p}\right)\xi \tag{12}$$

$$\frac{\partial^2 S_b}{\partial x^2} = -\left(\frac{3w_p G_s S_b}{Y_b w_b t_b t_p}\right)\xi \tag{13}$$

where
$$\xi = \left(\frac{S_p}{S_b} - 1\right) \tag{14}$$

Subtracting Eq. (13) from Eq. (12), we get

$$\frac{\partial^2 \xi}{\partial x^2} - \Gamma^2 \xi = 0 \tag{15}$$

where
$$\Gamma^2 = \left(\frac{G_s}{Y_p t_s t_p} + \frac{3G_s w_p}{Y_b w_b t_b t_p}\right) \tag{16}$$

This phenomenon of the difference in the PZT strain and the host structure's strain is called as ***shear lag effect***. The parameter Γ (unit m^{-1}) is called the ***shear lag parameter***. The ratio ξ is called as ***strain lag ratio***. The ratio ξ is a measure of the differential PZT strain relative to surface strain on the host substrate, caused by shear lag. The general solution for Eq. (15) can be written as

$$\xi = A\cosh\Gamma x + B\sinh\Gamma x \tag{17}$$

Since the PZT patch is acting as sensor, no external field is applied across it. Hence, free PZT strain = $d_{31}E_3 = 0$. Thus, following boundary conditions hold good:

(i) At x = -*l* , $S_p = 0 \Rightarrow \xi = -1$. (ii) At x = +*l*, $S_p = 0 \Rightarrow \xi = -1$.

Applying these boundary conditions, we can obtain the constants A and B as

$$A = \frac{-1}{\cosh \Gamma l} \qquad \text{and} \qquad B = 0 \tag{18}$$

Hence,
$$\xi = -\frac{\cosh \Gamma x}{\cosh \Gamma l} \tag{19}$$

Using Eq. (14), we can derive

$$\frac{S_p}{S_b} = \left(1 - \frac{\cosh \Gamma x}{\cosh \Gamma l}\right) \tag{20}$$

Fig. 3 shows a plot of the strain ratio (S_p/S_b) across the length of a PZT patch (l = 5mm) for typical values of Γ = 10, 20, 30, 40, 50 and 60 (cm^{-1}). From this figure, it is observed that the strain ratio (S_p/S_b) is less than unity near the ends of the PZT patch. The length of this zone depends on Γ, which in turn depends on the stiffness and thickness of the bond layer (Eq. 16). As G_s increases and t_s reduces, Γ increases, and as can be observed from Fig. 3, the shear lag phenomenon becomes less and less significant and the shear is effectively transferred over very small zones near the ends of the PZT patch.

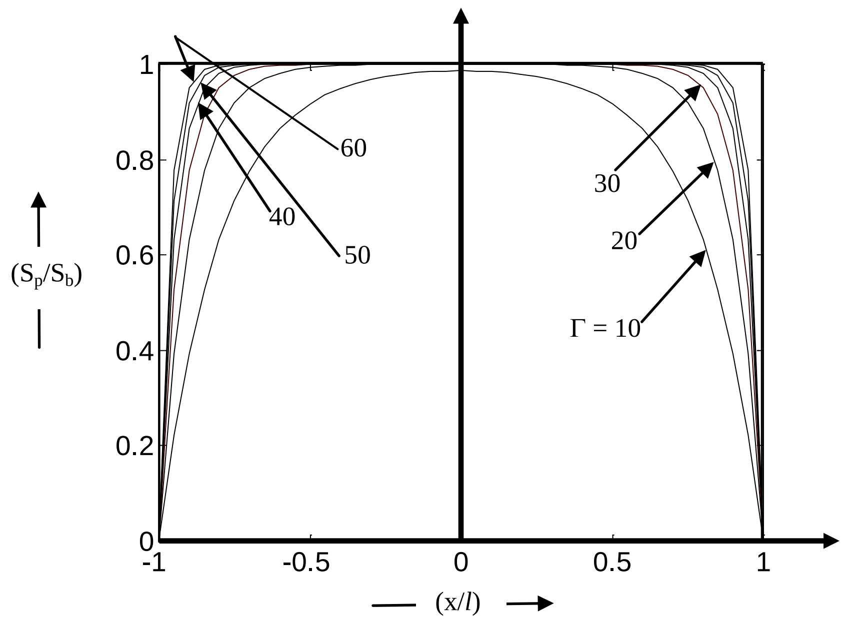


Fig. 3 Strain distribution across the length of PZT patch for various values of Γ.

Thus, if the PZT patch is used a sensor, it would develop less voltage across its terminals (than for perfectly bonded conditions) due to the shear lag effect. In other words, it will underestimate the strain in the substructure. In order to quantify the effect of shear lag, we can compute effective length of the sensor, as defined by (Sirohi and Chopra, 2000b)

$$l_{eff} = \int_{x=0}^{x=l} (S_p / S_b) dx \tag{21}$$

which is nothing but area under the curve (Fig. 7.3) between x = 0 and x = *l*. Hence, this is a sort of 'equivalent length', which could be deemed to have a constant strain, equal to S_b, the strain on the beam surface. Substituting Eq. (20) into Eq. (21) and upon integrating, we can derive effective length factor as

$$\frac{l_{eff}}{l} = 1 - \frac{\tanh \Gamma l}{\Gamma l} \tag{22}$$

Fig. 4 shows a plot of the effective length (Eq. 22) for various values of the shear lag parameter Γ. Typically, for $\Gamma > 30 cm^{-1}$, $(l_{eff} / l) > 93\%$, suggesting that shear lag effect can be ignored for relatively high ($> 30\ cm^{-1}$) values of Γ.

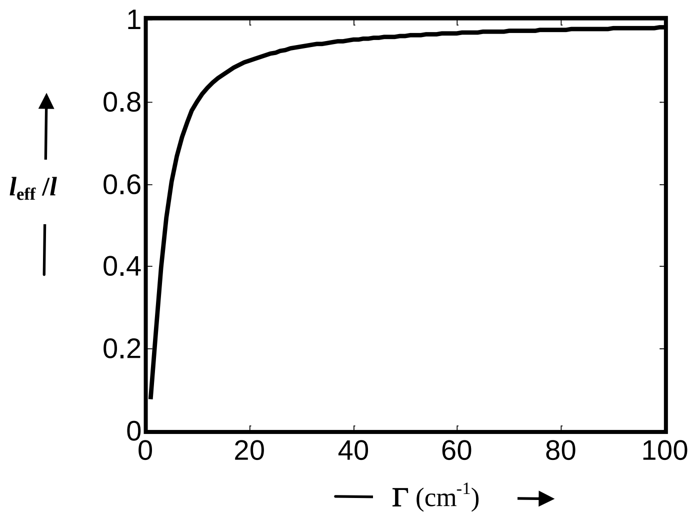


Fig. 4 Variation of effective length with shear lag factor.

PZT Patch as Actuator

If the PZT patch is employed as an actuator for a beam structure, it can be shown (Crawley and de Luis, 1987) that the strains S_p and S_b will be as given by

$$S_p = \frac{3\Lambda}{(3+\psi)} + \frac{\Lambda \psi \cosh \Gamma x}{(3+\psi)\cosh \Gamma l} \tag{23}$$

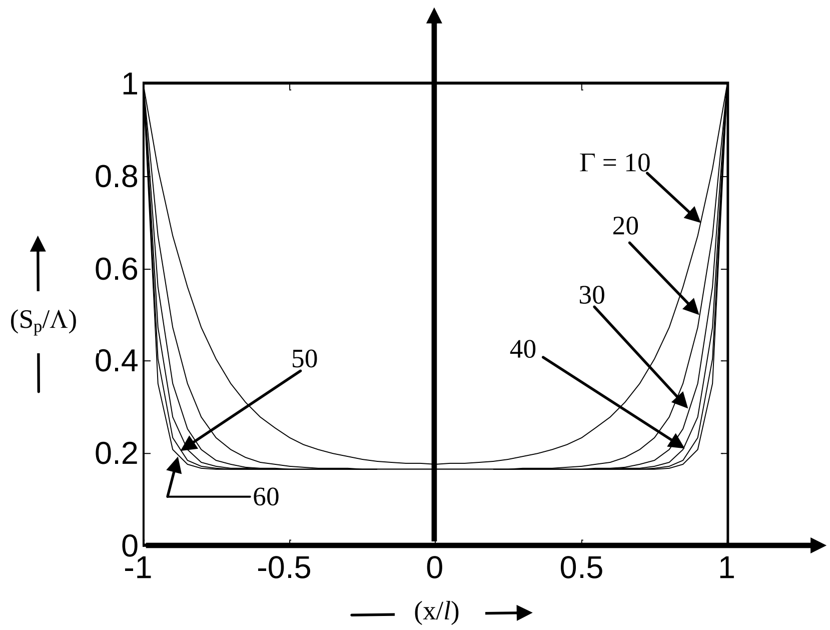
$$S_b = \frac{3\Lambda}{(3+\psi)} - \frac{3\Lambda \cosh \Gamma x}{(3+\psi)\cosh \Gamma l} \tag{24}$$

where $\Lambda = d_{31}E_3$ is the free piezoelectric strain and $\psi = (Y_b t_b / Y^E t_p)$ is the product of modulus and thickness ratios of the beam and the PZT patch. Fig. 5 shows the plots of (S_p / Λ) and (S_b / Λ) along the length of the PZT patch (l = 5mm) for $\psi = 15$. It is observed that like in the case of sensor, as Γ increases, the shear is effectively transferred over small zone near the two ends of the patch. As $\Gamma \to \infty$, the strain is transferred over an infinitesimal distance near the ends of the PZT patch. For the limiting case, as apparent from Fig. 5,

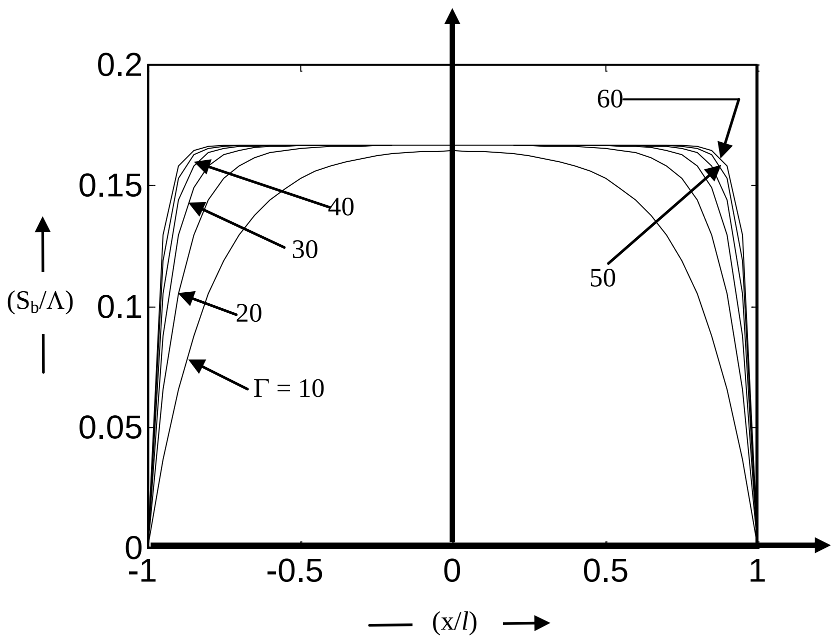
$$S_b = S_p = \frac{3\Lambda}{(3+\psi)} \tag{25}$$

which sets the maximum fraction of the piezoelectric free strain Λ that can be induced into the beam. Further, as $\psi \to 0$, $S_b \to \Lambda$.Typically, for $\Gamma > 30\text{cm}^{-1}$, the strain energy induced in the substructure by PZT actuator is within 5% of the perfectly bonded case. Therefore, for $\Gamma > 30\ \text{cm}^{-1}$, ignoring the effect of the bond layer will provide sufficiently accurate results for most engineering models.

It should be noted here that the analysis carried out by Crawley and de Luis (1987) as well as Sirohi and Chopra (2000b) is valid for static conditions only. These researchers extended their formulations to dynamic problems under the assumption that the operating frequency is small enough to ensure that the PZT patch acts ‘quasi-statically’. However, in the EMI technique, the operational frequencies are of the order of the resonant frequency of the PZT patch, warranting that the actuator dynamics should not be neglected.



(a)



(b)

Fig. 5 Distribution of piezoelectric and beam strains for various values of Γ. (a) Strain in PZT patch. (b) Beam surface strain.